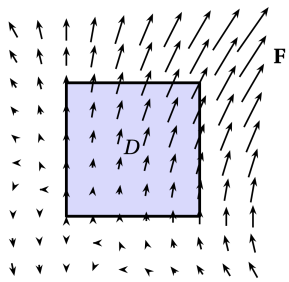
Math 203 — Practice with line integrals

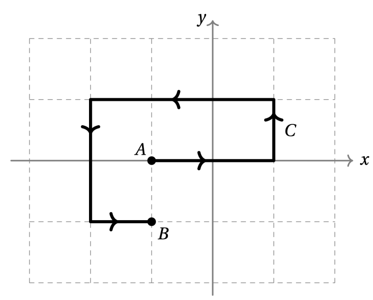
Problem 1. Let $\mathbf{F}(x,y) = \langle xy, x+y\rangle$, let C be the positively oriented square with vertices $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$, and let D be the region enclosed by C. See the image below.

a. Compute $\oint_C \mathbf{F}\cdot d\mathbf{r}$ by computing separate line integrals along the 4 sides of the square. To save time split the work up with a partner or two.

b. Compute $\iint_D \operatorname{curl}\mathbf{F}\, dA$ and compare with your previous answer. Do they match? Why?



Problem 2. Let $\mathbf{F}(x,y) = \left\langle x^3, 4x\right\rangle$, let C be the oriented curve from $A=(-1,0)$ to $B=(-1,-1)$ shown below. Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ using Green's Theorem. *Warning: C is not a closed curve. How can you get around this issue?*



Problem 3. Let $\mathbf{F}(x,y) = \left\langle 2xe^y, x + x^2e^y\right\rangle$, $\mathbf{G} = \langle 0, x\rangle$, and let C be the quarter circle oriented from $A=(4,0)$ to $B=(0,4)$. The image below shows $\mathbf{F}$ along with C.

a. Explain why $\mathbf{F}$ does not have a potential function.

b. Find a function f such that $\mathbf{F} = \mathbf{G} + \nabla f$.

c. Let C_1 be the line segment from $(0,0)$ to A and let C_2 be the line segment from $(0,0)$ to B. Find $\int_{C_1} \mathbf{G}\cdot d\mathbf{r}$ and $\int_{C_2} \mathbf{G}\cdot d\mathbf{r}$. These integrals can be done without computation and instead just thinking about how the vectors of $\mathbf{G}$ are related to the tangent vectors along C_1 and C_2.

d. Use Green's Theorem and part c. to compute $\int_C \mathbf{G}\cdot d\mathbf{r}$.

e. Use parts b. and d. to compute $\int_C \mathbf{F}\cdot d\mathbf{r}$.