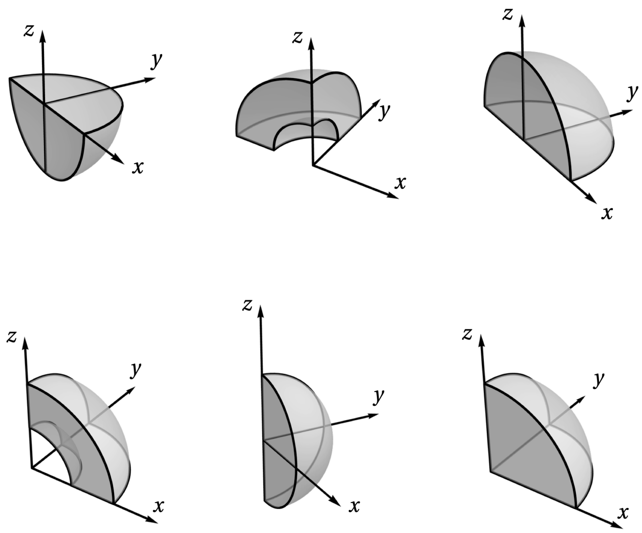
Math 203 — Exam 3 review

Your third exam will take place during the self-schedule exam period and will contain about 7 multi-part problems. It will cover material from Homework 7 to Homework 10 (in the textbook, this is material spanning sections 13.3-14.4, though not any sections we skipped. The problems below give you a sampling of some similar problems, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes. There are also problems in our textbook, with answers to odd-numbered problems in the back. No notes will be allowed on the exam, but there will be some formulas given on the exam sheet (eg. the conversion formulas for spherical coordinates) as well as the unit circle with corresponding values of sine and cosine. You'll be allowed to use a scientific calculator with no graphing or calculus functionality but the problems will be written so that a calculator is not required.

Problem 1. The figures below show different solid regions, either portions of the unit ball or portions of the solid between spheres of radius 1 and 2. Set up triple integrals for the volume of each solid using spherical coordinates.



Problem 2. Convert the following triple integrals from cylindrical to Cartesian coordinates or vice versa.

a. $\int_{-\pi/2}^{\pi/2}\int_0^2\int_0^{r^2} r\,dzdrd\theta$

b. $\int_0^{2\pi}\int_0^2\int_0^r r\,dzdrd\theta$

c. $\int_{-2}^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^2 xz\,dzdxdy$

d. $\int_{-3}^3\int_0^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2}\sqrt{x^2+y^2}\,dzdydx$

Problem 3. Convert the following double integrals from Cartesian to polar coordinates.

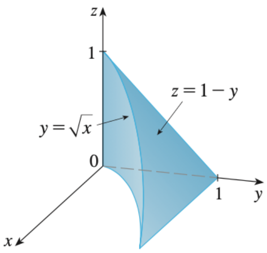
a. $\int_{-3}^3\int_0^{\sqrt{9-x^2}}\sin(x^2+y^2)\,dydx$

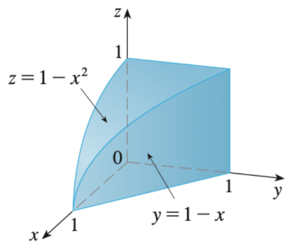
b. $\int_0^1\int_y^{\sqrt{2-y^2}}(x+y)\,dxdy$

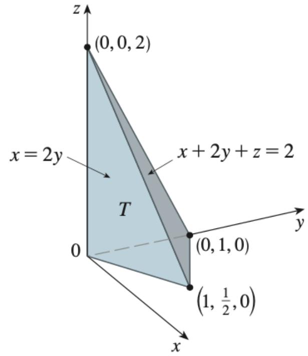
c. $\int_0^9\int_{-\sqrt{81-y^2}}^0 x^2y\,dxdy$

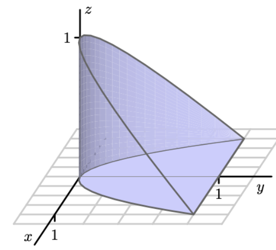
d. $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}\,dydx$

Problem 4. Setup triple integrals for each of the solids below using the following orders of integration: (1) $dV = dzdydx$, (2) $dV = dydzdx$, (3) $dV = dxdzdy$. Note the last solid is bounded by $y = x^2$, $z = 1 - y$, and $z = 0$.









Problem 5. For each vector field $\mathbf{F}$ below, determine whether it is conservative. If it is, find a potential function f and use it to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ for the given curve C. If it is not, compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ using a parametrization of C.

a. $\mathbf{F}(x, y) = \langle x^2, y^2 \rangle$, C is the arc of the parabola $y = 2x^2$ from $(-1, 2)$ to $(2, 8)$

b. $\mathbf{F}(x, y) = \langle ye^x + \sin y, e^x + x\cos y \rangle$, C is the quarter of the ellipse $x^2 + 2y^2 = 4$ from $(2, 0)$ to $(0, \sqrt{2})$

c. $\mathbf{F}(x, y) = \langle 2x - 2y, -3x + 4y - 8 \rangle$, C is the quarter of the unit circle from $(0, -1)$ to $(1, 0)$

d. $\mathbf{F}(x, y) = \langle ye^x + \sin y, e^x + x\cos y \rangle$, C is the unit circle oriented counterclockwise

e. $\mathbf{F}(x, y) = \langle xy^2, 2x^2y \rangle$, C is the line segment from $(1, 2)$ to $(3, 4)$

Problem 6. For each vector field $\mathbf{F}$ and oriented closed curve C given below, use Green's Theorem to compute $\oint_C \mathbf{F} \cdot d\mathbf{r}$. Be careful: check the orientation of the given curve when applying the theorem.

a. $\mathbf{F}(x, y) = \langle xy^2, 2x^2y \rangle$, C is the triangle oriented from $(0, 0)$ to $(0, 4)$ to $(2, 0)$ to $(0, 0)$

b. $\mathbf{F}(x, y) = \langle \cos y, x^2 \sin y \rangle$, C is the rectangle oriented from $(0, 0)$ to $(5, 0)$ to $(5, 2)$ to $(0, 2)$

c. $\mathbf{F}(x, y) = \left\langle y + e^{\sqrt{x}}, 2x + \cos y^2 \right\rangle$, C is the piecewise curve from $(0, 0)$ to $(1, 1)$ along $x = y^2$ and from $(1, 1)$ to $(0, 0)$ along $y = x^2$

d. $\mathbf{F}(x, y) = \langle \sin^3 x + 4y, 5x + \cos^2 y \rangle$, C is the circle $(x - 3)^2 + (y + 4)^2 = 4$ traced clockwise

Problem 7. For each vector field $\mathbf{F}$ and oriented curve C given below, set up $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$ using a parametrization of C. If possible, use the Divergence Theorem to compute $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$. Otherwise compute it using your parametrization

a. $\mathbf{F}(x, y) = \langle y^3, x^2 \rangle$, C is the circle $x^2 + y^2 = 9$ traced counterclockwise

b. $\mathbf{F}(x, y) = \langle \cos x, \sin y \rangle$, C is line segment from $(2, 0)$ to $(0, 2)$

c. $\mathbf{F}(x, y) = \langle x^2, y \rangle$, C is the piecewise closed curve from $(0, 1)$ to $(1, 0)$ along $y = 1 - x^2$, to $(0, 0)$ along $y = 0$, to $(0, 1)$ along $x = 0$

d. $\mathbf{F}(x, y) = \langle 0, x \rangle$, C is the line segment from $(1, 1)$ to $(5, 1)$