Math 206, Fall 2024 — Homework 2

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Due September 20 at 5:00 pm

Instructions. This problem set contains problems from Week 2 of class. The problem numbers refer to our textbook, *Reading, Writing, and Proving* by Ulrich Daepp and Pam Gorkin.

Problem 1. Do the following textbook problems: Problem 3.14, 3.15, 4.5 parts e, h, i, k (when negating the implications, first translate them into "for all" statements), 4.8, 5.4

Problem 2. State the contrapositive of each of the following statements.

1. If nm is even, then n is even or m is even.

2. If $x \in \mathbb{R}$ satisfies the equation $x^2 = x + 1$, then $x = 1 + \sqrt{5}$ or $x = 1 - \sqrt{5}$.

3. If $f(x)$ is a differentiable function, then it is a continuous function.

Problem 3. State the converse of each of the statements in the previous problem.

Problem 4. Suppose that x and y are positive real numbers. Prove that if $x < y$ then $x^3 < y^3$. *Advice: do some scratch work that involves working backwards from the conclusion before writing your proof. You may use the equation $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$ without justification.*

Problem 5. Consider the set
$$A = \left\{ \frac{p}{p+1} : p \in \mathbb{N} \right\}.$$

Prove that A does not have a largest element.