Math 206, Fall 2024 — Homework 5

Tim Chumley

Due October 18 at 5:00 pm

Instructions. This problem set contains problems from Weeks 5 and 6 of class. The problem numbers refer to our textbook, *Reading, Writing, and Proving* by Ulrich Daepp and Pam Gorkin.

Problem 1. Do the following textbook problems: Problems 9.4 (no justification needed), 9.5, 9.8, 9.16, 9.18, 9.20, 10.2beg (simply state which properties fail to hold, if any, for each example), 10.5, 10.8, 10.13

Problem 2. For each of the following subsets of $\mathbb{R}$, state whether it is (1) open but not closed, (2) closed but not open, (3) both open and closed, or (4) neither open nor closed. No justification is needed.

1. $\cup_{n=1}^{\infty}(0, 1 + 1/n)$

2. $\cup_{n=1}^{\infty}[0, 1 + 1/n]$

3. $\cap_{n=1}^{\infty}(0, 1 + 1/n)$

4. $\cap_{n=1}^{\infty}[0, 1 + 1/n]$

5. $\cup_{n=1}^{\infty}(n, n + 1)$

6. $\cup_{n=1}^{\infty}[n, n + 1]$

7. $\cap_{n=1}^{\infty}(n, n + 1)$

8. $\cap_{n=1}^{\infty}[n, n + 1]$

Problem 3. Let $n \in \mathbb{Z}^{+}$ and let $A_1, \ldots, A_n \subseteq \mathbb{R}$ be given closed sets. Prove that $\bigcup_{k=1}^{n} A_k$ is a closed set.