- Tim / Prof. / Prof. Chumley (he/him)
- Moodle – announcements, Q and A forum
- Webpage (tchumley.mtholyoke.edu/m206)
  - notes, worksheets, homework, syllabus
  - updated daily
- Homework (weekly)
  - submitted on Gradescope
  - due Fridays at 5 pm
  - redos (details to be announced)
- Quizzes – (mostly) weekly on Fridays, 15 minutes
- Exams – two during semester, one during finals, in-class
- Participation – come to class, be a good community member, stay in touch when something goes wrong (eg. illness)
- Office hours (tentative)
  - Mondays 11:15-12:15
  - Tuesdays 4:00-5:00
  - Thursdays 1:15-2:15

  drop in (Clapp 423), no appointment necessary

What is this class about?

- learning about mathematical reasoning and structures
- learning to read and communicate mathematics (largely by learning to read and write proofs)
- getting an introduction to the basics of analysis (real numbers, limits, sequences, continuous functions)

Question What is a mathematical proof? How should it look? What is the point of it? What do you anticipate might be hard about writing proofs?

Keywords:
- series steps involving deductive reasoning, starting from assumptions leading to conclusion
- complete sentences, paragraph form, important calculations on display
- point is to communicate, share knowledge, convince (yourself, others) something is true

Math 206 — Discussion handout

Below are three (informal) arguments, labeled *Note 1, Note 2 and Note 3*, for or against the claim that $1 = 0.999\ldots$. After reading the arguments, discuss the three questions below in a group of two or three.

Note 1. Observe that $1/3 = 0.333\ldots$. Multiply both sides by 3 to get $1 = 0.999\ldots$.

Note 2. Let $x = 0.999\ldots$. Multiply by 10 so that you have $10x = 9.999\ldots$. Now subtract x from both sides of this equation and solve for x:

$$\begin{aligned} 10x - x &= 9.999\ldots - x \\ 9x &= 9.999\ldots - 0.999\ldots \\ 9x &= 9 \\ x &= 1. \end{aligned}$$

Note 3. Actually, $1 \neq 0.999\ldots$. This is due to the fact that every real number has a unique decimal expansion.

Question 1. *Do you believe the claim is true or false? Do you believe any of the arguments? What assumptions are being made? What makes you wonder?*

Question 2. *Informally speaking, what do you think a mathematical proof is? How should it look? What is the purpose of proofs? What do you anticipate is hard about writing proofs?*

Question 3. *What are some things we can do in support of our learning and in support of a positive environment?*

<u>Definition</u> A (<u>logical</u>) <u>statement</u> is a sentence that can be determined to be true or false but not both

<u>Examples</u> Which of these are statements?

Which statements are true?

(a) Seven is a prime number.

(b) All odd numbers are prime.

(c) Hi, welcome to class.

(d) For each person in class, there is a date on which they were born.

(e) For each date, there is a person in class who was born on that date.

<u>Connectives</u> given a statement or statements we can form new (compound) statements using <u>connective</u> operators:

connective	name	English equivalent
$\neg P$	negation	"not P"
$P \wedge Q$	conjunction	"P and Q"
$P \vee Q$	disjunction	"P or Q"
$P \Rightarrow Q$	implication	"if P, then Q" or "Q when P" or "P only if Q"
$P \Leftrightarrow Q$	biconditional	"P if and only if Q"

antecedent

conclusion

Examples $P =$ "I like tomatoes" $Q =$ "I like pizza"

$R =$ "you clean your room", $S =$ "you can go to your friend's house."

$U =$ "3 is an even number" $W =$ "$2 > 1$"

Write the following in English sentences:

(a) $\neg U$ "3 is an odd number"

(b) $P \wedge Q$ "I like tomatoes and pizza"

(c) $P \vee Q$ "I like tomatoes or pizza"

(d) $R \Rightarrow S$ "If you clean your room, then you can go to your friend's house."

(e) $U \Rightarrow W$ "If 3 is an even number then $2 > 1$"

Truth tables: a way to summarize the possible true/false values of the statements above

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

$P \Leftrightarrow Q$ is defined to be $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

<u>Example</u> Make the truth table for $\neg(P \vee Q)$ and for $\neg P \wedge \neg Q$.

P	Q	$P \vee Q$	$\neg(P \vee Q)$	$\neg P$	$\neg Q$	$\neg P \wedge \neg Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

<u>Def</u> Two statements P and Q are <u>equivalent</u> if they have the same truth values. We write $P \Leftrightarrow Q$ when this happens.

<u>Example</u> negate the sentence "I like tomatoes or pizza"

"I do not like tomatoes and I do not like pizza"

"I like neither tomatoes nor pizza"