Math 206 — Closed sets

Problem 1. Make a conjecture about which of the following are closed sets. Right now, instead of writing a full formal proof, can you outline an argument explaining your conjecture?

a. $\emptyset$

b. $\mathbb{R}$

c. $\{1\}$

d. $[0, 1)$

e. $(-\infty, 0]$

f. $\bigcap_{n=1}^{10} [1 - 1/n, 1 + 1/n]$

g. $\bigcap_{n=1}^{\infty} [1 - 1/n, 1 + 1/n]$

Problem 2. Prove or disprove: the intersection of an infinite collection of closed sets is closed.

Problem 3. Prove or disprove: the union of an infinite collection of closed sets is closed.

Problem 4. Write a summary of what you know about finite and infinite collections of open sets. Are intersections of open sets always open? Are unions of open sets always open?

Problem 5. Repeat the previous problem with closed sets.