Math 206 — Power sets and Cartesian products

Problem 1. Let A be a set and $\mathcal{P}(A)$ its power set. Determine which of the following statements are true.

a. $A \in \mathcal{P}(A)$

b. $\emptyset \subseteq \mathcal{P}(A)$

c. $\emptyset = \mathcal{P}(\emptyset)$

d. $\{\emptyset\} = \mathcal{P}(\emptyset)$

e. If $a \in A$ then $\{a\} \subseteq \mathcal{P}(A)$

Problem 2. Consider the following expressions. In each, replace $\square$ by either the symbol $\in$ or the symbol $\subseteq$ to make a correct statement.

a. $\mathbb{N} \square \mathbb{Z}$

b. $\{1, 2, 3\} \square \mathbb{Z}$

c. $\{5\} \square \mathbb{Z}$

d. $\mathbb{Z}^+ \square \mathcal{P}(\mathbb{Z})$

e. $\{1, 2, 3\} \square \mathcal{P}(\{1, 2, 3\})$

f. $5 \square \mathbb{Z}$

g. $\emptyset \square \mathbb{Z}$

h. $\emptyset \square \mathcal{P}(\mathbb{Z})$

Problem 3. Give an informal geometric description of the following sets. Are they same set?

a. $\mathbb{Z} \times \mathbb{R}$

b. $\mathbb{R} \times \mathbb{Z}$

Problem 4. Let A be any set. Make a conjecture about how to simplify $A \times \emptyset$. Explain your simplification.

Problem 5. Let A, B, C, D be sets. The following statement is false

$$(A \cup C) \times (B \cup D) \subseteq (A \times B) \cup (C \times D).$$

Use your conjecture from Problem 4 to find a counterexample.