Math 206 — Relations

Problem 1. Define a relation on $\mathbb{Z}$ by $x \sim y$ if and only if $x^2 + y^2$ is even. Show that this is an equivalence relation. Give 5 examples of integers in the equivalence class of 1.

Problem 2. Consider the relation on $\mathbb{Z}$ given by $x \sim y$ if and only if $y - x$ is divisible by 7. It is possible to show that this is an equivalence relation. For each of $x = 0, 1, 2$ find give elements of E_x.

Problem 3. Define a relation on $\mathbb{R}$ by $x \sim y$ if and only if there exists $n \in \mathbb{Z}$ such that $x, y \in [n, n+1]$. Is this an equivalence relation? Why or why not?

Problem 4. Consider the following modification of the relation in the previous example. Define a relation on $\mathbb{R}$ by $x \sim y$ if and only if there exists $n \in \mathbb{Z}$ such that $x, y \in [n, n+1)$. This is in fact an equivalence relation. (Try proving it if you finish early.) What elements are in the equivalence class E_π? What elements are in the equivalence class of -1?