Math 206 — Order in the reals

Problem 1. Let $A \subseteq \mathbb{R}$ be a nonempty set. Answer the following true/false questions using definitions and intuition. No need to give a formal proof, but try to explain your reasoning for each.

a. An upper bound of A is an element of A.

b. The supremum of A is an element of A.

c. The set A has at least one maximum.

d. The set A has at most one maximum.

e. The set A has at least one upper bound.

f. The set A has at most one upper bound.

g. The set A has at least one supremum.

h. The set A has at most one supremum.

Problem 2. Consider the interval $A = (3, 4]$.

a. Prove that A does not have a minimum.

b. Prove that $\inf A = 3$.

c. What can you say about $\max A$ and $\sup A$?