Math 206 — More on completeness

Problem 1. Consider the set $S = \mathbb{R} \setminus \mathbb{Z}$. Give an example of a subset $T_1 \subseteq S$ that has an infimum and supremum in S and a bounded subset $T_2 \subseteq S$ that has neither an infimum nor supremum in S. *Note that the existence of such a set T_2 shows that S is not complete.*

Problem 2. Determine whether each of the following is true or false.

a. Let S be a subset of $\mathbb{R}$ consisting of 20 positive integers. Then S has a supremum and an infimum, both of which belong to S.

b. Suppose that S is a nonempty subset of $\mathbb{R}$ and S has a supremum U. Let $T = \{x \in S : x \leq U\}$. Then $T = S$.

c. Suppose that S is a nonempty bounded subset of $\mathbb{R}$ and $U = \sup S$. Suppose further that there exists $x \in S$ such that $x < U$. Let $v = \sup \{x \in S : x < U\}$. Then $v < U$.