Math 206 — More on logical statements

Problem 1. Consider the statements $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$. Make a truth table for both statements and state why you can conclude they're equivalent.

Problem 2. Consider the following statements. Make a truth table for each. What simpler statement are they both equivalent to?

   a. $P \vee (P \wedge Q)$

   b. $P \wedge (P \vee Q)$

Problem 3. The following are examples of implications (ie. statements of the form $P \implies Q$). Rephrase them so that they're in the form "if P then Q" and state which part is the hypothesis P and which part is the conclusion Q.

   a. The sum of two positive numbers is positive.

   b. The square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the two other sides.

   c. All primes are even.

   d. The square of every real number is positive.

Problem 4. Express the negation of each statement in the previous problem.

Problem 5. Just for fun, discuss whether the statement is true or the negation is true for each of the statements in Problems 3 and 4.

Problem 6. A *deductive argument* is a collection of statements $A_1, A_2, \ldots, A_n$ called *premises or hyptotheses* followed by a statement B called the *conclusion*. A deductive argument is called *valid* if whenever $A_1, A_2, \ldots, A_n$ are all true, B is true as well. In symbols, we can write a deductive argument as

$$A_1$$
$$A_2$$
$$\vdots$$
$$A_n$$
$$--$$
$$\therefore B$$

To check whether a deductive argument is valid, we can make a truth table using $A_1, \ldots, A_n$ and B and check that whenever $A_1, \ldots, A_n$ are all true, B is true as well. Make a truth table using the statements in the following argument and decide whether it is valid.

$$P \implies \neg Q$$
$$R \implies Q$$
$$R$$
$$--$$
$$\therefore \neg P$$