Math 206 — One-to-one and onto

Problem 1. For each of the following functions, determine whether it is one-to-one or onto or both. Explain your reasoning.

a. $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$ given by $f(x) = 1/x$

b. $f : \mathbb{Z} \times (\mathbb{Z} \times \setminus \{0\}) \to \mathbb{R}$ given by $f(x, y) = x/y$

c. $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2 + 4x + 5 = (x + 2)^2 + 1$

Problem 2. Suppose we are given a function $f : A \to B$ but we are actually interested in studying the function on only a subset $C \subseteq A$ of the domain. We can then define a new function, called the ***restriction of f to C***, which we denote by $f|_C : C \to B$ and is given by $f|_C(x) = f(x)$ for any $x \in C$. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ and a nonempty subset $C \subseteq \mathbb{R}$ such that f is not one-to-one but $f|_C$ is one-to-one.

Problem 3. Give an example of a function $f : (0, \infty) \to \mathbb{R}$ that is a bijection or explain why such a function cannot exist. Can you give an example of a bijection $g : \mathbb{R} \to (0, \infty)$?