Math 206 — Inverses

Problem 1. Let $f : A \to B$ be a bijection. Prove that f^{-1} is a bijection.

Problem 2. Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3 - 5$. It can be shown that f is a bijection. Let's take this as given for this problem. Use algebra to determine f^{-1}. Note that using part 4 of today's theorem, you can check your work by computing $f \circ f^{-1}$ or $f^{-1} \circ f$.

Problem 3. We have previously shown that $f : \mathbb{Z} \to \mathbb{N}$ given by

$$f(n) = \begin{cases} 2n & n \geq 0 \\ -2n - 1 & n < 0 \end{cases}$$

is a bijection. Find f^{-1}. Note that using part 4 of today's theorem, you can check your work by computing $f \circ f^{-1}$ or $f^{-1} \circ f$.

Problem 4. Consider the functions $f : \mathbb{R} \setminus \{-2\} \to \mathbb{R} \setminus \{1\}$ and $g : \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{-2\}$ given by

$$f(x) = \frac{x-3}{x+2}, \quad g(x) = \frac{3+2x}{1-x}.$$

a. Calculate $f \circ g$ and $g \circ f$.

b. Can we use the previous part and part 4 of today's theorem to conclude anything?