Math 206 — More Induction

Problem 1. For which values of $n \in \mathbb{N}$ is the inequality $n^2 \geq n+1$ true? Make a conjecture and prove it using induction.

Problem 2. Prove that $n! > n^2$ for all $n \in \mathbb{N}$ such that $n \geq 4$.

Problem 3. Prove that

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

for any $n \in \mathbb{Z}^+$.

Problem 4. Consider the recursively defined sequence $(T_n)_{n=0}^{\infty}$ defined by $T_0 = 1$ and

$$T_{n+1} = 1 + \sum_{k=0}^{n} T_k$$

for all $n \in \mathbb{N}$.

a. Calculate T_1, T_2, T_3, T_4.

b. Make a conjecture for a closed-form (ie. non-recursive) formula for T_n and prove your formula using strong induction.

Problem 5. Consider the Fibonacci sequence $(F_n)_{n=0}^{\infty}$ defined by $F_0 = 0, F_1 = 1$ and

$$F_{n+2} = F_{n+1} + F_n$$

for all $n \in \mathbb{N}$. Let $a = (1+\sqrt{5})/2$ and $b = (1-\sqrt{5})/2$ be solutions to the equation $x^2 = x + 1$. Prove using strong induction that for all $n \in \mathbb{N}$,

$$F_n = \frac{a^n - b^n}{a - b}.$$