Math 206 — Sequences

Problem 1. State the infimum of each of the following sequences. No justification needed.

a. $x_n = 1/n$ for $n \in \mathbb{Z}^+$

b. $x_n = n^2$ for $n \in \mathbb{N}$

c. $x_n = n/(n+1)$ for $n \in \mathbb{N}$

d. $x_n = (-1)^n/(n^2+1)$ for $n \in \mathbb{N}$

Problem 2. A sequence (x_n) is **increasing** if $x_n \leq x_{n+1}$ for all n and **decreasing** if $x_n \geq x_{n+1}$ for all n. It is **strictly increasing** if $x_n < x_{n+1}$ for all n and **strictly decreasing** if $x_n > x_{n+1}$ for all n. For each of the following give an example of a sequence that meets the given constraints or explain why it is not possible.

a. A bounded sequence. Find an upper bound and a lower bound.

b. A sequence that is bounded below but not bounded above. Find a lower bound. Does your example need to be an increasing sequence?

c. A sequence that is bounded above but not bounded below. Find an upper bound.

d. An increasing sequence that is neither bounded above nor below.

e. A strictly increasing bounded sequence.

f. A strictly decreasing sequence that is bounded above but not below.

g. A sequence that is neither strictly increasing nor strictly decreasing.

Problem 3. Let (x_n) be a sequence that is bounded below. Let $L = \inf(x_n)$. Define $y_n = -x_n$ for all n. Prove that (y_n) is bounded above, make a conjecture about the value of $\sup(y_n)$, and prove your conjecture.