

Math 206 — Contrapositive, converse, and first proofs

Problem 1. For each of the following statements, write out the contrapositive and converse statements. Discuss whether either is true.

- If you are the President of the United States, then you live in a white house.
- If you are going to bake a souffle, then you need eggs.
- If x is a real number, then x is an integer.
- If x is a real number, then $x^2 < 0$.

Problem 2. Consider the statement “if x is odd, then $x^2 + 3x + 1$ is odd.”

- State the contrapositive.
- Prove either the statement itself or its contrapositive.
- State the converse.
- Explain why the converse is not true by giving a counterexample.

Problem 3. Consider the statement “if x and y are real numbers, then $x^2 + y^2 \geq 2xy$.” This is a true statement and our goal in this problem will be to write a proof. It might feel hard to start because the assumptions (that x and y are real numbers) are so open ended. Sometimes it helps to do some scratch work and generate ideas by manipulating the conclusion and working backwards from the conclusion. Try doing that here and then write your proof. Make sure your proof starts from the given assumptions and then reaches the desired conclusion.

Problem 4. Suppose x and y are positive real numbers. Prove that if $x < y$ then $x^2 < y^2$. Use an approach like in the previous problem.