Math 206 — Monotone Convergence Theorem

Problem 1. Suppose that (x_n) is a decreasing sequence and is bounded below. Prove that (x_n) converges. *Start by making a conjecture for the value of the limit.*

Problem 2. Let $x_0 = 1$ and define $x_n = (x_{n-1}+1)/3$ for all $n \in \mathbb{Z}^+$. Prove that (x_n) is decreasing and bounded below by $1/2$. Explain why (x_n) converges and compute its limit.

Problem 3. Let $x_0 = \sqrt{3}$ and define $x_n = \sqrt{x_{n-1}+3}$ for all $n \in \mathbb{Z}^+$. Prove that (x_n) converges using the Monotone Convergence Theorem and compute its limit (you may use without proof the fact that if $x_n \to L$ and $L > 0$ then $\sqrt{x_n} \to \sqrt{L}$).