Math 206 — Operations on Sets

Problem 1. For each positive integer n, define

$$A_n = [0, 1/n), \quad B_n = [0, 1/n], \quad C_n = (0, 1/n).$$

Find the following:

a. $\bigcup_{n=1}^{\infty} A_n, \bigcup_{n=1}^{\infty} B_n, \bigcup_{n=1}^{\infty} C_n$

b. $\bigcap_{n=1}^{\infty} A_n, \bigcap_{n=1}^{\infty} B_n, \bigcap_{n=1}^{\infty} C_n$

Problem 2. Simplify the following set:

$$\bigcap_{j \in \mathbb{Z}} (\mathbb{R} \setminus (j, j+1)).$$

Problem 3. Let I be a given index set and let $\{A_\alpha : \alpha \in I\}$ be an indexed collection of sets. Prove that for all $\beta \in I$

$$\bigcap_{\alpha \in I} A_\alpha \subseteq A_\beta \subseteq \bigcup_{\alpha \in I} A_\alpha.$$

Problem 4. Let I be a given index set and let $\{A_\alpha : \alpha \in I\}$ be an indexed collection of sets. Prove that if $A \subseteq A_\alpha$ for all $\alpha \in I$ then

$$A \subseteq \bigcap_{\alpha \in I} A_\alpha.$$