

Math 241, Spring 2026 — Homework 1

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Due February 5 at 5:00 pm

Instructions. This problem set covers material from Week 1 of class. The problem numbers refer to our textbook, *A First Course in Chaotic Dynamical Systems*, by Robert L. Devaney.

Problem 1. Please do the following exercises from Chapter 3: 2, 3, 5, 6, 7, 8

Remark 1. In Exercise 3, please expand your answers out and simplify. Your answers should look like polynomials of the form $a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ for some $n \geq 1$ and some coefficients a_0, a_1, \dots, a_n .

Remark 2. In Exercise 5, you do not need to prove your general formula for $F^n(x)$. Just recognizing the pattern is ok.

Remark 3. In Exercise 6, you might find it helpful to compute $A(x_0), A^2(x_0), A^3(x_0)$ for a few values like $x_0 = -2, -1, 0, 1, 2$.

Remark 4. In Exercise 7, make sure to know the definition of a fixed point, which is given on page 27. This problem comes down to solving some equations. Remember you can do this algebraically but sometimes it's also helpful to think geometrically.

Remark 5. In Exercise 8, make sure to know the definition of an eventually fixed point, which is given on page 29.

Problem 2. Consider the function $f(x) = x^2 - 2$ and let $x_0 \neq 0$ be a given real number.

- Give a formula for the tangent line of the graph of f at the point $(x_0, f(x_0))$. Your formula should be in the form $y = mx + b$ for some $m, b \in \mathbb{R}$ in terms of x_0 .
- Let x_1 be the value along the x -axis where the tangent line in part (a) crosses. Find the value of x_1 in terms of x_0 . In other words, find a function $F(x)$ such that $x_1 = F(x_0)$.
- Suppose $x_0 = 1$. Use MATLAB to find the first 5 iterates of the function F you found in part (b). List them here. What value do you think the orbit converges to?
- Was the choice of $x_0 = 1$ special? Do you always converge to the same value regardless of the choice of initial seed? Try a few different choices of x_0 and explain what you see. Make sure to try both positive and negative numbers for your initial seed. Make sure to try more than just 5 steps in the orbit. Explain your observations in complete sentences.
- What questions do you have about this algorithm?

Problem 3. Read Chapter 1 of our textbook. Please respond to the following with a few sentences for each part.

- What is a Julia set? In the figures of the various Julia sets, what do the black points represent? What do the colored points represent?
- In the brief history discussed through Section 1.2, what were you most interested to learn more about? Why?