Math 241, Spring 2026 — Homework 2

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Due February 12 at 5:00 pm

Instructions. This problem set covers material from Week 2 of class. The problem numbers refer to our textbook, *A First Course in Chaotic Dynamical Systems*, by Robert L. Devaney.

Problem 1. Please do the following exercises from Chapter 3: 10, 12

Remark 1. In Exercise 12, you should find all possible prime period 2 points. Your work will involve solving $F^2(x) = x$ by factoring.

Problem 2. Please do the following exercises from Chapter 4: 2, 4

Remark 2. In Exercise 2, this notation, which can also written as

$$\left\{ x_0 \in \mathbb{R} : \lim_{n\to\infty} F^n(x) = \pm\infty \right\},$$

means the set of all initial seeds whose orbits go to $\pm\infty$. Your answers should be written as intervals or unions of intervals.

Remark 3. In Exercise 4, you should use cobweb plots to do this. You may use Desmos but please make sure that you feel comfortable with drawing cobweb plots by hand. Note that performing a complete orbit analysis means finding the long-term behavior of the orbit for every initial seed in phase space. Please give your answer to each part of this problem as a bullet-point breakdown of all possible cases. Please also include a cobweb plot with at least one example orbit drawn by hand for each part.

Problem 3. Let $D : [0, 1) \to [0, 1)$ be the doubling map. Recall that

$$D(x) = \begin{cases} 2x & 0 \le x < 1/2 \\ 2x - 1 & 1/2 \le x < 1. \end{cases}$$

a. Give the formula for $D^2(x)$. Hint: you will have to give a piecewise definition like the one given for $D(x)$ and your definition will have 4 pieces to it.

b. Give the formula for $D^3(x)$. Hint: you will have to give a piecewise definition like the one given for $D(x)$ and your definition will have 8 pieces to it.

c. Plot the graphs of $y = D^2(x)$ and $y = D^3(x)$.

d. Use your graphs to explain how many period 2 and period 3 points the map D has.

Problem 4. Let $D : [0, 1) \to [0, 1)$ be the doubling map. Suppose that $x_0 = \frac{k}{2^m - 1} \in [0, 1)$ for some $k, m \in \mathbb{N}$. Show that x_0 is periodic with period m.

Remark 4. Note that $x_0 = 1/3 = 1/(2^1 - 1)$, $x_0 = 1/7 = 1/(2^3 - 1)$, and $x_0 = 2/9 = 14/(2^6 - 1)$ are all examples of seeds of this form that we saw in our doubling map worksheet.