

Math 241, Spring 2026 — Homework 7

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Due April 2 at 5:00 pm

Instructions. This problem set covers material from Week 9 of class. Any chapter and exercise numbers refer our textbook, *A First Course in Chaotic Dynamical Systems*, by Robert L. Devaney.

Problem 1. Please do the following textbook problems from Chapter 7: 3, 4, 6, 10, 22, 23

Problem 2. In constructing the Cantor Middle-Thirds set, we proceeded iteratively where at each step of the process, we removed the middle-third of each interval that remained after the removal process of the previous step. Find the total length of the intervals that were removed over the course of this process. Informally speaking, what do you find strange about this result?

Problem 3. Consider the following 8 elements of the Cantor Middle-Thirds set Γ :

$$1/3, 2/3, 1/9, 2/9, 7/9, 8/9, 1/27, 2/27.$$

These are called endpoints of Γ because they are endpoints of the middle-thirds intervals that get removed.

- a. For each of them, find the ternary expansion that contains only 0's and 2's.
- b. Based on your work in the previous part, make a conjecture for the general structure of the ternary expansions of endpoints of Γ .