Math 241, Spring 2026 — Homework 9

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Due April 16 at 5:00 pm

Instructions. This problem set covers material from Week 11 of class. Any chapter and exercise numbers refer our textbook, *A First Course in Chaotic Dynamical Systems*, by Robert L. Devaney.

Problem 1. Please do the following textbook exercises from chapter 9: 1, 2, 5, 8.

Problem 2. A dynamical system $F : X \to X$ is said to be *transitive* if for each $x, y \in X$ and each $\epsilon > 0$ there exists $z \in X$ and $N \in \mathbb{N}$ such that $d(x, z) < \epsilon$ and $d(F^N(z), y) < \epsilon$. Moreover, the dynamical system is said to have a *dense orbit* if there exists $x_0 \in X$ such that the set $\{F^n(x_0) : n \geq 0\}$ is dense in X. Show that if a dynamical system has a dense orbit then the dynamical system is transitive.

Problem 3. For each $x \in \Sigma$ below, find $y \in \Sigma$ such that $d(x, y) < 1/100$ and y is an eventually fixed point of the shift map σ.

a. $x = (\overline{0})$.

b. $x = (\overline{01})$

Problem 4. Show that the set of eventually fixed points of the shift map is dense in Σ.

Problem 5. Suppose we have two dynamical systems $F : X \to X$ and $G : Y \to Y$ defined on different phase spaces X and Y . Moreover, suppose that there is an invertible function $h : X \to Y$ with the property that $h \circ F \circ h^{-1} = G$ (meaning for all $y \in Y$ we have $h(F(h^{-1}(y))) = G(y)$). Suppose that $x_0 \in X$ is a periodic point of F with period n. Show that $h(x_0)$ is a periodic point of G with period n.