- Tim / Prof. / Prof. Chumley (he/him)
- Moodle - announcements
- Webpage (tchumley.mtholyoke.edu/m241)
  - notes, worksheets, homework, syllabus
  - updated daily
- Homework (weekly)
  - written problems, submitted on Gradescope
  - due Thursdays at 5 pm
- Quizzes - Wednesdays (first one is Feb 4)
- Exams - two during semester
- Participation - come to class, be a good community member, stay in touch when something goes wrong (eg. illness)
- Office hours (tentative)
  - Mondays 4:00-5:00
  - Wednesdays 4:00-5:00
  - Thursdays 11:00-12:00

  drop in (Clapp 423), no appointment necessary

Chapters 1-3 Introduction to Dynamical Systems

Informally, a dynamical system is a mathematical model that describes the evolution of some phenomenon that changes over time (e.g. a population size, stock price, a fluid particle).

Def A discrete-time dynamical system is a set X called the phase space or state space together with a function $F : X \longrightarrow X$ called a map that describes the step-by-step rule of evolution.

domain "inputs"

codomain "possible outputs"

Remark ① Our class will be focused on discrete-time systems. Continuous-time systems are studied in Math 333 Differential Eq's.

② Our class will focus on the case when $X \subseteq \mathbb{R}$ "real numbers" or possibly $X \subseteq \mathbb{C}$. "complex numbers"

"subset"

Given a dynamical system and an initial state x_0 called the seed, the evolution is defined by repeatedly applying F to get the next state in time:

$$x_1 = F(x_0)$$
$$x_2 = F(x_1) = F(F(x_0))$$
$$x_3 = F(x_2) = F(F(F(x_0)))$$
$$\vdots$$
$$x_n = F(x_{n-1}) = (F \circ F \circ \cdots \circ F)(x_0)$$

"composition"

$$= F^n(x_0)$$ (our notation for n-times composition).

The infinite sequence of states $x_0, x_1, x_2, \ldots$ that gets generated is called the <u>orbit of x_0</u> and x_n is called the <u>n^{th}</u> <u>iterate</u> of x_0.

<u>Example</u> A simple model of population dynamics says that the population in the succeeding generation is directly proportional to the size of the current generation:

$$F(x) = rx$$

for some constant r (called a <u>parameter</u>) determined by ecological conditions. Then

$$x_1 = F(x_0) = r x_0$$
$$x_2 = F(x_1) = r^2 x_0$$
$$x_3 = F(x_2) = r^3 x_0$$
$$\vdots$$
$$x_n = r^n x_0.$$

so $F^n(x) = r^n x$

It's rare to be able to find closed form expression of F^n.

When $r > 1$, the population grows to infinity, when $r < 1$, the population goes extinct, and when $r = 1$, the population remains constant.

This semester we'll often study families of dynamical systems indexed by a parameter. Our aim will be to understand how the long-term qualitative behavior depends on this parameter.

Example Consider the system where $X = \mathbb{R}$ and

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{5}{x_n}\right).$$

Here, $F(x) = \frac{1}{2}\left(x + \frac{5}{x}\right)$. The orbit of, for example, $x_0 = 1$, looks like

$$x_1 = \frac{1}{2}(1+5) = 3$$

$$x_2 = \frac{1}{2}\left(3 + \frac{5}{3}\right) = \frac{7}{3} = 2.3333\ldots$$

$$x_3 = \frac{1}{2}\left(\frac{7}{3} + \frac{15}{7}\right) = \frac{47}{21} = 2.2381\ldots$$

$$x_4 = 2.236068\ldots$$

$$x_5 = 2.236067\ldots$$

$$x_6 = 2.236067\ldots$$

It converges to $\sqrt{5}$! We can view this as an algorithm for numerically approximating $\sqrt{5}$. Does it work for other initial seeds? Why does it work.

See Section 2.5 for some intuition.