Chapter 4 Graphical Analysis

Our goal today is to visualize orbits with the aim of understanding more about long-term behavior.

A cobweb diagram is a plot of

① $y = F(x)$

② $y = x$

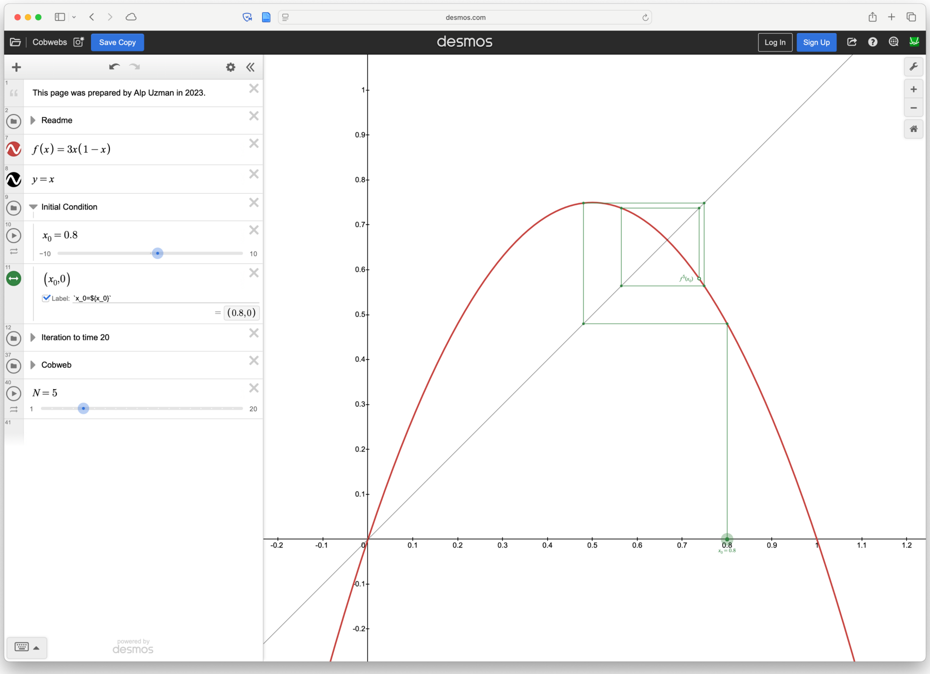
③ the points $(x_0, F(x_0))$, $(F(x_0), F(x_0))$,

$(F(x_0), F^2(x_0))$, $(F^2(x_0), F^2(x_0))$,

$\ldots$, $(F^{n-1}(x_0), F^n(x_0))$, $(F^n(x_0), F^n(x_0))$, $\ldots$

connected by directed line segments.

It visualizes the orbit of a seed x_0 under F.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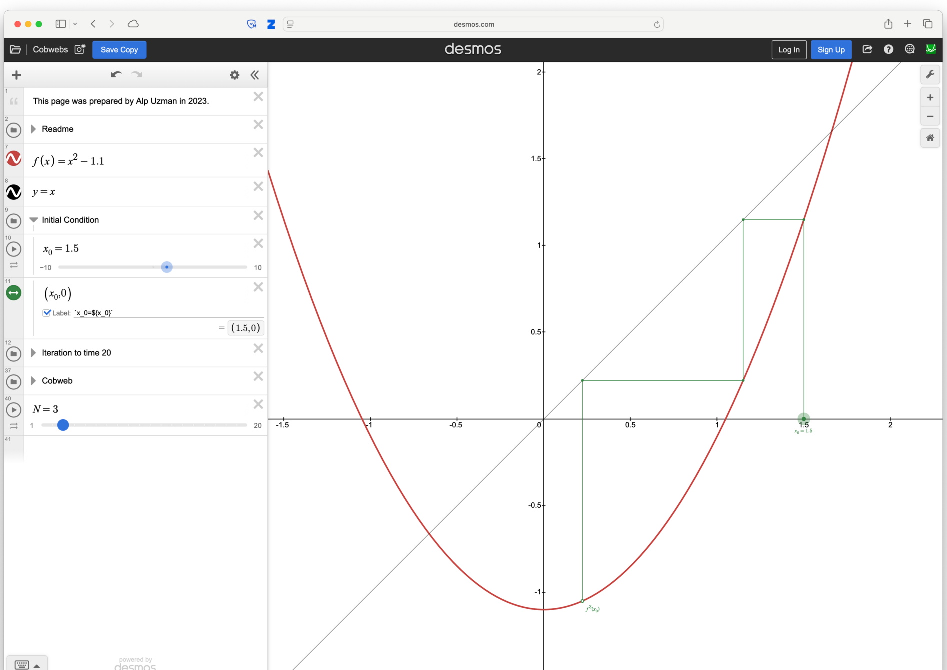
Readme
$f(x) = 3x(1-x)$
$y = x$
Initial Condition
$x_0 = 0.8$
$(x_0, 0)$
Label: `x_0=${x_0}`
$= (0.8, 0)$
Iteration to time 20
Cobweb
$N = 5$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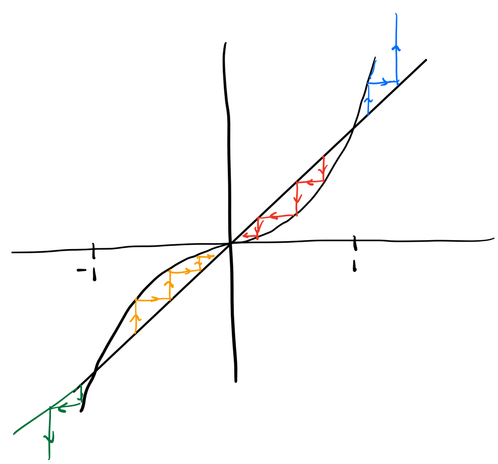
This page was prepared by Alp Uzman in 2023.
Readme
$f(x) = x^2 - 1.1$
$y = x$
Initial Condition
$x_0 = 1.5$
$(x_0, 0)$
Label: `x_0=${x_0}`
$= (1.5, 0)$
Iteration to time 20
Cobweb
$N = 3$

A complete <u>orbit analysis</u> describes the long term behavior of <u>every</u> initial seed $x_0 \in X$. It's not always possible to do this with cobwebs and graphical analysis (and these are not a formal, rigorous argument) but here's an example of what we mean.

<u>Example</u> Let $X = \mathbb{R}$ and $F : X \to X$ given by $F(x) = x^3$. Give a complete orbit analysis using cobwebs.



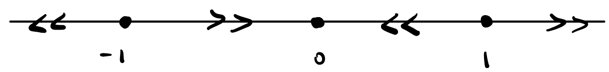
The fixed points are $x_0 = -1, 0, 1$.

When $x_0 \in (1, \infty)$, $\lim_{n \to \infty} F^n(x_0) = \infty$.

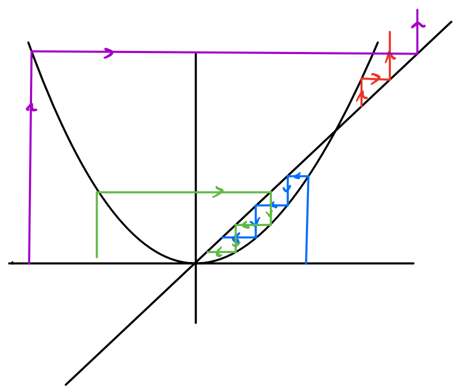
When $x_0 \in (-1, 1)$, $\lim_{n \to \infty} F^n(x_0) = 0$

When $x_0 \in (-\infty, -1)$, $\lim_{n \to \infty} F^n(x_0) = -\infty$.

A phase portrait shows the behavior of orbits on the real line:



Example Repeat with $F(x)=x^2$. Include eventual fixed points.



The fixed points are $x_0 = 0, 1$, and $x_0 = -1$ is an eventual fixed point.

When $|x_0| > 1$, $\lim_{n\to\infty} F^n(x_0) = \infty$

When $|x_0| < 1$, $\lim_{n\to\infty} F^n(x_0) = 0$.