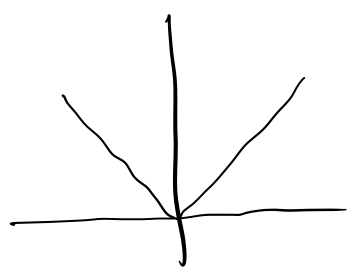
Discussion of Homework

Exercise 3.8 What are the eventually fixed points of $A(x) = |x|$.

Recall x_0 is an eventually fixed point if some element of its orbit $\{x_0, x_1, x_2, \ldots\}$ is a fixed point.



what is the orbit of x_0 when $x_0 \geq 0$?

what is the orbit of x_0 when $x < 0$?

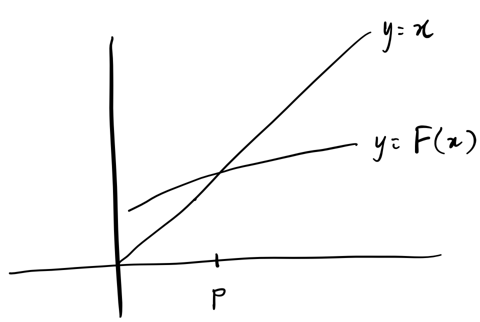
Chapter 5

Previously we discussed, informally, the ideas of

attracting fixed points: if our initial seed is near an attracting fixed point it converges to it

repelling fixed points: if our initial seed is near a repelling fixed point, the orbit diverges away from it.

Discussion Suppose p is a fixed point and $0 < F'(p) < 1$. Let $x_0 < p$ be a nearby initial seed.



Notice - the graph of $F(x)$ is

above $y=x$ when $x<p$ and

below $y=x$ when $x>p$

(at least near p)

- if $x<p$, the picture shows us

$$x < F(x) < p$$

- This means if $x_0 < p$ and $x_1 = F(x_0)$,

then $x_0 < x_1 < p$

- We could repeat this argument with

x_1 and $x_2 = F(x_1)$:

$$x_1 < x_2 < p$$

- It seems that we're getting

$$x_0 < x_1 < x_2 < \cdots < x_n < p$$

for any n.

Where is this leading? What happens as $n \to \infty$? It seems,

we have a sequence that's always growing but can't grow above p.

What must happen?

We must have that $x_n \to p$ as $n \to \infty$!

Monotone Convergence Theorem

A bounded, monotone sequence of real numbers must converge.