§5.5 Periodic Points

Def Let x_0 be a periodic point with prime period n. We say its corresponding n-cycle orbit is

1. attracting if $|(F^n)'(x_0)| < 1$
2. repelling if $|(F^n)'(x_0)| > 1$
3. neutral if $|(F^n)'(x_0)| = 1$

It can be hard to compute F^n explicitly so what do we do if we need to compute $(F^n)'(x_0)$?

Chain rule $(F \circ G)'(x) = F'(G(x))G'(x)$.

2-cycle if x_0 is a prime period 2 point, then

$$(F^2)'(x_0) = (F \circ F)'(x_0)$$
$$= F'(F(x_0))F'(x_0)$$
$$= F'(x_1)F'(x_0)$$

3-<u>cycle</u> if x_0 is a prime period 3 point, then

$$(F^3)'(x_0) = (F \circ F^2)'(x_0)$$

$$= F'(F^2(x_0)) \cdot (F^2)'(x_0)$$

$$= F'(x_2) \cdot F'(x_1) \cdot F'(x_0)$$

<u>Example</u> Let $F(x) = -\frac{3}{2}x^2 + \frac{5}{2}x + 1$. Then $F(0) = 1$, $F(1) = 2$, and $F(2) = 0$. So 0 lies on a 3-cycle. Also $F'(x) = -3x + \frac{5}{2}$. Therefore

$$|(F^3)'(0)| = |F'(0)||F'(1)||F'(2)|$$

$$= \left(\frac{5}{2}\right)\left(\frac{1}{2}\right)\left(\frac{7}{2}\right) = \frac{35}{8} > 1$$

and so the 3-cycle is repelling

<u>n-cycle</u> when x_0 is a prime period n point

$$(F^n)'(x_0) = F'(x_n)\, F'(x_{n-1}) \cdots F'(x_1)\, F'(x_0).$$