

Math 241, Spring 2022 — Introduction

Class on January 25

Problem 1. Suppose a population of a certain species has a maximum possible size, and if the population ever reaches that maximum size, it will die out in the next generation. Let P_n denote the fraction of the maximum size that's alive in generation n . Suppose that

$$P_{n+1} = 2P_n(1 - P_n).$$

This population model is called the *logistic model* and the value 2 is a parameter of the system that I've chosen somewhat arbitrarily for the purposes of the introductory questions below.

1. What are the *state space* X and *map* F of this system?
2. Suppose the population size P_n is small in generation n . Roughly speaking, what happens to the population size P_{n+1} in generation $n + 1$?
3. Suppose the population size P_n is big in generation n . Roughly speaking, what happens to the population size P_{n+1} in generation $n + 1$?
4. Using a calculator, what happens to the population after 10 generations when $P_0 = 0.2$? When $P_0 = 0.7$? List the *orbit* of each of these initial values.
5. What do you think the term *fixed point* means?
6. Do you think the system has any fixed points? How many? What do you think they are?
7. What equation does a fixed point have to satisfy? Try using algebra to show that the system has 2 fixed points and to find what they are.