Math 241, Spring 2026 — Introduction

Problem 1. Suppose the population of a certain ecological species has a maximum possible size M and, rather than studying the population size in absolute terms, we wish to study its size relative to M. We will use what is known as the *logistic model*:

$$x_{n+1} = \lambda x_n(1 - x_n),$$

where $\lambda > 0$ is a given constant and x_n represents the population size divided by M in generation n. For the sake of this problem, suppose that $\lambda = 2$.

a. What are the phase space X and the map F in this dynamical system?

b. Suppose that the population size in generation in n is very small relative to M.

   1. What is the value of $1 - x_n$ roughly?
   2. Given your previous answer, in terms of x_n, what is the value of x_{n+1} roughly?

c. Suppose that the population size in generation in n is very close to M.

   1. What is the value of $1 - x_n$ roughly?
   2. Given your previous answer, what is the value of x_{n+1} roughly?

d. Suppose the seed is $x_0 = 0$. What is orbit of x_0?

e. Repeat the previous question with $x_0 = 0.5$.

f. What do you think the term *fixed point* means? Trying completing the following sentence: an element $x_0 \in X$ is a fixed point if its orbit is....

g. What equation does a fixed point have to satisfy? (Take a look at page 27 of the textbook after class to check whether your idea for the equation is correct.) Try solving your proposed equation to show that the system has 2 fixed points.

h. Suppose the seed is $x_0 = 0.2$. List the next 10 elements of the *orbit* of x_0. Repeat this using $x_0 = 0.9$. Please list the orbit elements to 4 decimal places. Do this with a calculator in class. Modify lines 12-14 of the iterator.m script and run the file in MATLAB after class to automate this process.

i. Run the time_series.m file in MATLAB to generate a visualization of your previous answer. Please submit the image with your answer to this problem.

j. We will soon learn the terms *attracting fixed point* and *repelling fixed point*. Based on your previous answers, do you think either of the fixed points you found in part (g) is attracting? Repelling?