Math 241, Spring 2022 — Bifurcations in the logistic map

Class on March 1

In this set of problems we continue our exploration of bifurcations, now using the logistic map

$$F_\lambda(x) = \lambda x(1-x).$$

Problem 1. Consider the equation $F_\lambda(x) = x$.

1. Find solutions to this equation using algebra.
2. For which values of λ does the logistic map have only one fixed point? Two?
3. Use Desmos to visualize the bifurcation at $\lambda = 1$ that is hinted at in your previous answers. We say a *saddle node bifurcation* occurs at λ_0 when the system goes from 0 to 1 to 2 fixed points as we vary λ around λ_0, and the graph of $F_{\lambda_0}(x)$ is tangent to the line $y = x$. Do we have a saddle node bifurcation in this example?

Problem 2. Suppose $\lambda \neq 0, 1$. In the previous problem you should have found that 0 and $\frac{\lambda-1}{\lambda}$ are the two fixed points of the system.

1. For which values of λ is 0 attracting? Neutral? Repelling?
2. For which values of λ is $\frac{\lambda-1}{\lambda}$ attracting? Neutral? Repelling?

Problem 3. Let's move on to investigating period doubling.

1. Use Desmos and the graphs of F_λ and F_λ^2 to make a conjecture about values of λ when period-2 points first appear.
2. What do you notice about the slopes of F_λ and F_λ^2 at fixed points when a period-doubling bifurcation occurs?
3. What seems to be the relationship between neutral fixed points and period-doubling bifurcations?
4. Use Desmos to investigate the appearance of period-4 points. When do they appear? What can you say about the slopes of F_λ^2 and F_λ^4?