

Math 241— The Cantor Middle-Thirds Set

Problem 1. Find the real number represented by the ternary expansion $0.00222\bar{2}$.

Problem 2. Find the ternary expansion of $1/13$.

Problem 3. Fill in the blank in the following true statement: each element of the Cantor middle-thirds set has a unique ternary expansion consisting of only the digits ____.

Problem 4. Is $1/4$ an element of Γ ? What about $1/13$? Why or why not?

Problem 5. We know that $2/3$ is an element of Γ ; it never gets removed during the process of constructing Γ by removing open middle-thirds subintervals. You can check that it has ternary expansion $2/3 = 0.122\bar{2}$. Why is this not in contradiction to what you said in Problem 3?

Problem 6. The values $1/9, 2/9, 1/27$ are examples of elements of Γ that came about as endpoints of the intervals in Γ_n defined last time. Show that each has two different ternary expansions.