Math 241— The doubling map in binary

Problem 1. Find the orbit of $x_0 = 5/12$ under the doubling map D. First compute it using fractions as usual and then write out the binary expansion of each element of the orbit. You can find the binary expansion of a number using a method similar to how we found ternary expansions.

Problem 2. You probably have started to notice that if $x_0 = 0.a_1a_2a_3\ldots$ in binary, then $D(x_0) = 0.a_2a_3a_4\ldots$ in binary. Let's try to explain why by breaking things down into two cases.

a. Suppose the binary expansion of x_0 is $0.0a_2a_3a_4\ldots$. Then

$$x_0 = \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \cdots$$

What is the binary expansion of $2x_0$? What is the binary expansion of $D(x_0)$?

b. Suppose the binary expansion of x_0 is $0.1a_2a_3a_4\ldots$.

$$x_0 = \frac{1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \cdots$$

What is the binary expansion of $2x_0$? What is the binary expansion of $D(x_0)$ in this case? Hint: the binary expansion of $2x_0$ and $D(x_0)$ will be different (why?).

Problem 3. Let $n \geq 1$. Suppose $x_0 = 0.a_1a_2a_3\ldots$ in binary. Write out the binary expansion of $D^n(x_0)$.

Problem 4. Using what you have learned in the previous problems

a. Find the binary expansion of all period 2 points.

b. Do the same for period 3 points.

c. Let $n \geq 1$ be arbitrary. Explain how many period n points there are.