

Math 241— Itineraries

Problem 1. Consider the tent map

$$T(x) = \begin{cases} 3x & x \leq 1/2 \\ 3 - 3x & x > 1/2. \end{cases}$$

Let Γ denote the Cantor Middle-Thirds set and let $I_0 = [0, 1/3]$ and $I_1 = [2/3, 1]$. Note that $\Gamma \subset I_0 \cup I_1$. Suppose that $x_0 \in \Gamma$. Then $T^n(x_0) \in \Gamma$ for all $n \geq 1$. Therefore $T^n(x_0) \in I_0 \cup I_1$ for all $n \geq 1$. Let $S(x_0) = (s_0, s_1, s_2, \dots)$ where

$$s_n = \begin{cases} 0 & \text{if } T^n(x_0) \in I_0 \\ 1 & \text{if } T^n(x_0) \in I_1. \end{cases}$$

- Find $S(1/3)$, $S(1/4)$, and $S(3/10)$.
- Bonus: find a period 2 point x_0 such that $S(x_0) = (0, 1, 0, 1, 0, 1, \dots) = (\overline{01})$.
- Bonus: find a period 3 point x_0 such that $S(x_0) = (0, 0, 1, 0, 0, 1, \dots) = (\overline{001})$.

Problem 2. Consider the doubling map $D(x) = 2x \pmod{1}$. Let $I_0 = [0, 1/2)$ and $I_1 = [1/2, 1)$. Let $S(x_0) = (s_0, s_1, s_2, \dots)$ where

$$s_n = \begin{cases} 0 & \text{if } D^n(x_0) \in I_0 \\ 1 & \text{if } D^n(x_0) \in I_1. \end{cases}$$

Find $S(1/3)$, $S(2/3)$, and $S(5/12)$. How do these compare with the binary expansions of $1/3$, $2/3$, and $5/12$?