

Math 241, Spring 2022 — Ternary expansions

Class on March 31

In this set of problems we continue to get familiar with the Cantor Middle Thirds set and ternary expansions.

Problem 1. Let $x = \frac{1}{26}$ and suppose that its ternary expansion is $0.s_1s_2s_3\dots$

1. Which interval does x fall into: $[0, 1/3]$, $(1/3, 2/3)$, or $[2/3, 1]$? What is the value of s_1 ?
2. Which interval does $x - \frac{s_1}{3}$ fall into: $[0, 1/9]$, $(1/9, 2/9)$, or $[2/9, 1/3]$? What is the value of s_2 ?
3. Which interval does $x - \frac{s_1}{3} - \frac{s_2}{9}$ fall into: $[0, 1/27]$, $(1/27, 2/27)$, or $[2/27, 1/9]$? What is the value of s_3 ?
4. Answer similar questions to find s_4, s_5, s_6 . What pattern do you notice?
5. Compute $\sum_{i=1}^{\infty} \frac{s_i}{3^i}$ using your conjecture for the general pattern. What does this computation tell you?
6. Is x in the Cantor Middle Thirds set?

Problem 2. What if $x = 2/243$? Can you find a ternary expansion? This will be a little easier since $243 = 3^5$. Is it in the Cantor Middle Thirds set?

Problem 3. What if $x = 1/243$? Can you find a ternary expansion? Is it in the Cantor Middle Thirds set?