

Math 241— Sequence space and shift map

Problem 1. Let $x = (1, 1, 1, 1, 0, 0, \dots) = (1111\bar{0})$ and $y = (1, 1, 1, \dots) = (\bar{1})$.

- Find $d(x, y)$, $d(\sigma(x), \sigma(y))$, and $d(\sigma^4(x), \sigma^4(y))$.
- What is $d(\sigma^n(x), \sigma^n(y))$ for all $n \geq 5$?
- Complete the following statement, which is the moral of the story in this problem:
 x and y are close to each other, but ...

Problem 2. Let $x = (\bar{0})$. Find an example $y \in \Sigma$ such that $d(x, y) < 1/2^5$ and $d(\sigma^7(x), \sigma^7(y)) = 2$.

Problem 3. Repeat the previous problem except use $x = (\bar{01})$.

Problem 4. Suppose that I give you some arbitrary $x \in \Sigma$ and $\epsilon > 0$. Explain how to find $y \in \Sigma$ and $N \in \mathbb{N}$ such that $d(x, y) < \epsilon$ and $d(\sigma^N(x), \sigma^N(y)) = 2$.

Problem 5. Suppose that I give you some arbitrary $x \in \Sigma$ and $\epsilon > 0$. Explain how to find a periodic point $y \in \Sigma$ such that $d(x, y) < \epsilon$.

Problem 6. (Bonus) A map $F : \Sigma \rightarrow \Sigma$ is said to be continuous at $x_0 \in \Sigma$ if for each $\epsilon > 0$ there exists a corresponding $\delta > 0$ such that: if $y \in \Sigma$ and $d(x_0, y) < \delta$ then $d(F(x_0), F(y)) < \epsilon$. Show that σ is continuous at every $x_0 \in \Sigma$.