

Math 206 — Exam 2 review guide

Your exam in class on April 22 will contain about 5 problems, some with multiple parts. You should expect to see a question or two asking you to state some definitions or answer some true false questions where it is important to know definitions. You should also expect a few questions in the style similar to worksheets, homework, and quizzes. It will cover material from Homework 6 to Homework 9. In the textbook, this is material spanning Chapters 7, 9, and 10. I have outlined some important definitions, theorems, and general topics below. Also, the problems below give you a sampling of some problems like those that will appear on the exam, but it's not necessarily comprehensive, so make sure to review old homework, quizzes, worksheets, and lecture notes.

Definitions

While not necessarily comprehensive, here is an absolutely-must-know list of definitions and statements.

- Cantor middle-thirds set, countable set, uncountable set, ternary expansion of a real number, binary expansion of a real number
- sequence space on 2 symbols, shift map, Proximity Theorem
- sensitive dependence on initial conditions, dense set, transitive dynamical system, chaotic dynamical system, conjugate dynamical systems

Sample problems

These problems are not comprehensive and there are more than you will see on the exam itself, but they are meant to give you a general idea of the kinds of questions to expect.

Problem 1. The following questions are about ternary expansions and the map

$$T(x) = \begin{cases} 3x & x \leq 1/2 \\ 3 - 3x & x > 1/2. \end{cases}$$

- Find real numbers whose ternary expansions are:
 - $0.001\bar{0}$
 - $0.00\bar{2}$
 - $0.0\bar{1}0\bar{1}$
 - $0.\bar{2}0$
- Are any of the above elements of the Cantor middle-thirds set Γ ? Are any of them endpoints of Γ ?
- Find a ternary expansion of each prime period 2 point of T .
- Suppose $x \in \Gamma$ with ternary expansion $0.a_1a_2a_3\dots$. What is the ternary expansion of $T(x)$? Note that you'll have to consider two cases. In one of the two cases it will be helpful to note that $\sum_{n=1}^{\infty} 2/3^n = 1$.

Problem 2. The following questions are about the sequence space on two symbols

$$\Sigma = \{(x_0, x_1, x_2, \dots) : x_n \in \{0, 1\} \text{ for all } n \geq 0\}$$

with distance function d given by

$$d(x, y) = \sum_{n=0}^{\infty} \frac{|x_n - y_n|}{2^n}$$

and the shift map $\sigma : \Sigma \rightarrow \Sigma$ given by $\sigma(x_0, x_1, x_2, \dots) = (x_1, x_2, \dots)$.

- a. Find the distance $d(x, y)$ when
 1. $x = (\overline{100}), y = (\overline{010})$
 2. $x = (\overline{1011}), y = (\overline{01})$
- b. Find the two sequences which are midway between $x = (\overline{0})$ and $y = (\overline{1})$. In other words, find z_1 and z_2 such that $d(x, z_i) = d(y, z_i) = d(x, y)/2$ for $i = 1, 2$.
- c. Which element of Σ is furthest from the sequence $(s_0 s_1 s_2 \dots)$?
- d. Let $x = (\overline{0})$. Find $y \in \Sigma$ which is an eventually fixed point of σ such that $d(x, y) \leq 1/2^3$.
- e. Let $x = (\overline{0})$. Find $y \in \Sigma$ and $N \in \mathbb{N}$ such that $d(x, y) < 1/2^3$ and $d(\sigma^N(x), \sigma^N(y)) = 2$.

Problem 3. State whether each of the following is true or false.

- a. The set $Y = \{(s_0, s_1, s_2, s_3, \dots) \in \Sigma : s_2 = 0\}$ is dense in Σ .
- b. Consider $x = 9/10 \in \Gamma$. The itinerary of x is just its ternary expansion with each digit divided by 2.
- c. Suppose $F : X \rightarrow X$ and $G : Y \rightarrow Y$ are conjugate dynamical systems with conjugacy $h : X \rightarrow Y$. If $x_0 \in X$ is an eventually fixed point of F then $h(x_0)$ is an eventually fixed point of G .
- d. The shift map has 30 prime period 5 points.