

## Math 241, Spring 2026 — Doubling Map

The doubling map  $D : [0, 1) \rightarrow [0, 1)$  is given by

$$D(x) = \begin{cases} 2x & 0 \leq x < 1/2 \\ 2x - 1 & 1/2 \leq x < 1 \end{cases}$$

and it outputs the fractional value of  $2x$  for each  $x \in [0, 1)$ .

**Problem 1.** Sketch the graph of  $y = D(x)$  and find where it intersects with  $y = x$ . How many fixed points does the doubling map have?

**Problem 2.** What is the orbit of  $x_0 = 1/3$ ? What about  $x_0 = 1/7$  or  $x_0 = 2/9$ ? What name do we call these orbits?

**Problem 3.** What is the orbit of  $x_0 = 1/2$ ? What about  $x_0 = 1/4$  or  $x_0 = 3/16$ ? These kinds of initial seeds have a special name. What is it?

**Problem 4.** What is the orbit of  $x_0 = 1/6$ ? What about  $x_0 = 1/10$  or  $x_0 = 5/12$ ? What should we call these orbits?

**Problem 5.** Download and open the file [doubling\\_map.m](#) from the class web page. Try running it. Note that the default initial seeds are for orbits we computed together. You can try some different initial seeds as well. What do you notice? Is the output what you expect? In rough terms, why is the program behaving this way?

**Problem 6.** Going back to Problems 2, 3, and 4, what patterns do you notice? Can you make some conjectures? What kinds of initial seeds seem to produce periodic orbits? Eventually fixed orbits? Why?