Math 241, Spring 2022 — Attraction, repulsion, and derivatives

Class on February 8

In this set of problems we continue to get practice understanding the behavior of orbits that start near a fixed point.

Problem 1. For each of the following functions, (1) find its fixed points, (2) the value of $F'(x)$ at each of the fixed points, and (3) draw the graph of $F(x)$ and use a cobweb diagram to determine the behavior of orbits that start near a fixed point. Summarize the behavior in words and with a phase diagram.

1. $F(x) = x^2 - \frac{x}{2}$
2. $F(x) = 2.5x - x^2$

Problem 2. Suppose p is a fixed point of $F(x)$ and suppose $F'(p)$ is **positive** and **less than 1**. Make a sketch in the xy-plane of the curves $y = x$ and $y = F(x)$ that depicts this scenario, zooming in on their intersection.

1. Suppose x_0 is an initial seed that is bigger than p. As you make a cobweb diagram, what can you say about $x_1 = F(x_0)$? Does it get closer to p or farther? What about $x_2 = F^2(x_0)$, $x_3 = F^3(x_0)$ and so on? Informally, why is this happening? What is it about the slope of F?
2. Suppose we have an initial seed x_0 that is smaller than p. What happens here?
3. What general principle seems to be happening? Can you formulate it as a complete sentence?

Note. What we're doing in this worksheet captures a glimpse at what it's like to do mathematics. We start with analyzing concrete examples, then we try to formulate a general principle that's occuring. Finally, we try to make a convincing argument that combines established facts, calculations, and a narrative that ties everything together. Sometimes this process takes many iterations but an important lesson is that mathematics often involves making progress in small steps.