Math 241— The Quadratic Map

Problem 1. Let $Q_c : \mathbb{R} \to \mathbb{R}$ be given by $Q_c(x) = x^2 + c$ where $c \in \mathbb{R}$ is a given constant. We call c a *parameter* of the dynamical system.

a. For which values of c does Q_c have no fixed points? Two fixed points? One fixed point? Start by solving $Q_c(x) = x$ and when there are two solutions, call p_+ the bigger one and p_- the smaller one.

b. Make a graph of $y = Q_c(x)$ and $y = x$ in each of these three cases.

c. When Q_c has one fixed point p, is p attracting, neutral, and repelling? You should be able to answer this using the pictures you drew above, but check it using $|Q_c'(p)|$ as well.

d. Suppose we have chosen c from the range of values so that Q_c has two fixed points.

   1. When is p_+ repelling? Always? Never? For some subset of c values?
   2. What about p_-? When is it attracting? Repelling? Neutral?
   3. Use Desmos and cobweb plots to check your work to these questions.

Problem 2. By solving the equation $Q_c^2(x) = x$ using algebra, we can find that

$$q_\pm = \frac{1}{2}\left(-1 \pm \sqrt{-4c-3}\right)$$

are solutions.

a. For which values of c does Q_c fail to have any 2-cycles?

b. For which values of c does Q_c have an attracting 2-cycle? What connections do you see to the problem above?