Math 241, Spring 2022 — Saddle node bifurcation of $Q_c(x)$

Class on February 22

In this set of problems we get introduced to the *saddle node bifurcation* of the family of maps $Q_c(x) = x^2 + c$. When this bifurcation takes place, our dynamical system transitions from having no fixed points to one fixed point (the c value when this happens is called the *bifurcation value*) to two fixed points.

Problem 1. The following set of questions gets you to find the bifurcation value and formulas for the fixed points of Q_c when they exist.

1. Use the quadratic formula to find solutions to the equation $Q_c(x) = x$.
2. For which range of values of c does this equation have no real valued solutions? One solution? Two?
3. When Q_c has one fixed point, what is its value?
4. Consider the range of values where Q_c has two fixed points. Which is the one on the left and which is on the right when looking at a graph of $Q_c(x)$ and $y = x$?

Problem 2. The following quesitons get you to characterize the fixed points as attracting, repelling, or neutral. Let p_- be the left fixed point and p_+ be the right fixed point when the system has two fixed points.

1. Find $Q_c'(x)$.
2. For which values of c is p_+ attracting, repelling, or neutral? Does this reconcile with what you see graphically?
3. Answer these previous two questions for p_-. Be careful with the algebra in this case since it's a little trickier. Remember that the inequality $|x| < 1$ is equivalent to the two sided inequality $-1 < x < 1$ and the direction of an inequality changes when both sides are multiplied by a negative number.

Problem 3. Write a summary of your findings.

1. State the range of c values when there are no fixed points.
2. State the range of c values when there is one fixed point, state its value, and state whether it is attracting, repelling, or neutral.
3. State the range of c values when there are two fixed points. For each, state the range of c values when they are attracting, repelling, or neutral.