

Math 301, Fall 2021 — Homework 5

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Due October 8

Instructions. Please submit your solutions to the following problems on Gradescope. For the group part, please type your answers in LaTeX and submit the output PDF. You'll only submit one write-up for the whole group. For the solo part, you may handwrite solutions or use LaTeX. Make sure to select which problem is on each page in Gradescope.

Group problem

No group problem this week.

Solo problems

Problem 1. Consider the following sequences:

$$w_n = (-2)^n, \quad x_n = 5^{(-1)^n}, \quad y_n = 1 + (-1)^n, \quad z_n = n \cos\left(\frac{n\pi}{4}\right).$$

Find the lim sup and lim inf of each.

Problem 2. Let (s_n) be the sequence of numbers in Figure 11.2 of our textbook, listed in the indicated order.

1. Find the set S of subsequential limits of (s_n) .
2. Find $\limsup s_n$ and $\liminf s_n$.

Problem 3. Consider the sequences $(s_n), (t_n)$ given below, which repeat in cycles of four:

$$(s_n) = (0, 1, 2, 1, 0, 1, 2, 1, 0, 1, 2, 1, \dots)$$
$$(t_n) = (2, 1, 1, 0, 2, 1, 1, 0, 2, 1, 1, 0, \dots).$$

Find

1. $\liminf s_n + \liminf t_n$
2. $\liminf(s_n + t_n)$
3. $\limsup s_n + \limsup t_n$
4. $\limsup(s_n + t_n)$

Problem 4. Let (s_n) and (t_n) be bounded sequences.

1. Prove that for any $N \geq 1$

$$\sup \{s_n + t_n : n > N\} \leq \sup \{s_n : n > N\} + \sup \{t_n : n > N\}.$$

2. Prove that $\limsup(s_n + t_n) \leq \limsup s_n + \limsup t_n$.

Problem 5. Prove the (s_n) is bounded if and only if $\limsup |s_n| < \infty$.

Problem 6. Consider a sequence (s_n) .

1. Prove that $\lim s_n = 0$ if and only if $\lim |s_n| = 0$.

2. Prove that $\limsup |s_n| = 0$ if and only if $\lim s_n = 0$.