Math 301 — Quiz 1

Summary. Please do the following problems. You may use your book, notes, and other class materials, but you may not consult with each other or use any other resources. You may use LaTeX or handwrite your solutions. Please do whichever you prefer.

Problem 1. We showed in class that ordered space $X = \mathbb{Q}$ fails to have the least upper bound property using the set $S = \{r \in \mathbb{Q} : r^2 < 2\}$ as our counterexample. Explain why an ordered space given by an open interval, like $X = (0, 1)$, fails to have the least upper bound property.

Problem 2. Suppose that A and B are nonempty, bounded subsets of $\mathbb{R}$ and suppose that $\sup A \leq \sup B$.

a. Let $x \in A \cup B$. Show that $x \leq \sup B$.

b. Show that $\sup(A \cup B) \leq \sup B$.

c. Show that $\sup(A \cup B) = \sup B$.