

Math 301 — Liminf and limsup

Problem 1. Let $a_n = (-1)^n$ and let $u_n = \inf \{a_k : k > n\}$, $v_n = \sup \{a_k : k > n\}$ for all $n \geq 1$.

- Find u_1, u_2, u_3 .
- Find v_1, v_2, v_3 .
- What do you think you can conclude about $\liminf_{n \rightarrow \infty} a_n$ and $\limsup_{n \rightarrow \infty} a_n$?

Problem 2. Repeat Problem 1 using $(a_n) = (1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, \dots)$.

Problem 3. Repeat Problem 1 using $a_n = (-1)^n/n$ but compute the first 6 terms of (u_n) and (v_n) .

Problem 4. Repeat Problem 1 using $a_n = (-1)^n + (-1)^n/n$ but compute the first 6 terms of (u_n) and (v_n) .

Problem 5. Suppose $A \subseteq B$. How do $\inf A$ and $\inf B$ compare, ie. which is bigger? What about $\sup A$ and $\sup B$?

Problem 6. Consider the sequences (u_n) and (v_n) defined above.

- Are they increasing or decreasing?
- Why do they converge when (a_n) is bounded?
- What if (a_n) is bounded above but not bounded below?
- What if (a_n) is bounded below but not bounded above?