# Math 301 — Series

*Summary.* As we go through and prove the series tests that arise in second semester calculus, it's also useful to go back and review how they're used to prove that certain examples converge or diverge. The following question asks you to use a variety of the tests we've gone through so far.

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**Problem 1.** For each of the following series, decide whether it converges, giving a justification based on one of the tests we've introduced along with the convergence of our baseline examples (geometric series and $p$-series). These are the warm-up problems

a. $\sum \frac{\cos^2 n}{n^2}$

b. $\sum \frac{1}{2^n+n}$

c. $\sum \frac{1}{\ln n}$

**Problem 2.** Here are some more examples of the same flavor. Again, decide whether each converges and give justification.

a. $\sum \frac{n}{n+1}$

b. $\sum \frac{n^2}{n^3-3}$

c. $\sum \frac{1}{n!}$

Problem 1 All examples use comparison test here.

a $\frac{\cos^2 n}{n^2} \leq \frac{1}{n^2}$ for all $n$.

Thus $\sum \frac{\cos^2 n}{n^2}$ converges since $\sum \frac{1}{n^2}$ converges

(b) $\frac{1}{2^n + n} \leq \frac{1}{2^n}$ for all $n$

Then $\sum \frac{1}{2^n+n}$ converges since

$\sum \frac{1}{2^n}$ converges

(c) $\frac{1}{\ln n} > \frac{1}{n}$ for all $n$ since

$e^n > n$, which implies $n > \ln n$

Therefore $\sum \frac{1}{\ln n}$ diverges since $\sum \frac{1}{n}$

diverges.

Problem 2

(a) $\lim \frac{n}{n+1} = 1 \neq 0$, so $\sum \frac{n}{n+1}$

diverges by Test for Divergence.

(b) $\frac{n^2}{n^3 - 3} > \frac{n^2}{n^3} = \frac{1}{n}$ for all $n \geq 2$

since $n^3 - 3 < n^3$ for all $n \geq 2$

Therefore $\sum \frac{n^2}{n^3-3}$ diverges by

comparison test since $\sum \frac{1}{n}$ diverges.

(c) $\frac{1}{n!} < \frac{1}{n^2}$ for all $n \geq 4$

since $n^2 < n!$ for all $n \geq 4$.

Therefore $\sum \frac{1}{n!}$ converges since $\sum \frac{1}{n^2}$

converges.