Math 301 — Basic inequalities

Summary. So much of continuous math is centered on giving bounds on how close two objects, like real numbers, are. Throughout real analysis, we'll work with inequalities that allow us to talk about limits, approximation, and estimates. The inequalities we consider below will soon feel like second nature, and in subsequent work, you may use them without proof.

Problem 1. Recall the triangle inequality that we proved together: $|a+b| \leq |a|+|b|$ for any $a, b \in \mathbb{R}$.

a. Prove that for any $a, b, c \in \mathbb{R}$, $|a+b+c| \leq |a|+|b|+|c|$. Can you use the triangle inequality above?

b. What general statement can you make for a sum of n numbers?

Problem 2. Let $a > 0$ be a positive number that's given to us. Consider the set $\{x \in \mathbb{R} : |x| \leq a\}$.

a. What is an upper bound for this set? A lower bound?

b. Can you write the inequality $|x| \leq a$ as two inequalities involving x?

c. Make a sketch on the number line for the set of real numbers that satisfy $|x| \leq 1$.

d. Can you answer these questions when we consider $|x-3| \leq 1$?

Problem 3. Suppose x is a real number that satisfies the inequality $|x| < \epsilon$ for every positive real number $\epsilon > 0$. What do you think we can conclude about x? What if $|x-3| < \epsilon$ for all $\epsilon > 0$?