

## Math 301 — Some preliminaries

**Problem 1.** Let  $a, b, c, d \in \mathbb{R}$ . Use the triangle inequality to give brief proofs for the following inequalities.

- a.  $|a - b| \leq |a| + |b|$
- b.  $|a + b + c| \leq |a| + |b| + |c|$
- c.  $|a - b| \leq |a - c| + |c - d| + |d - b|$

**Problem 2.** Decide which of the following statements is true. Give a brief justification if the statement is valid and a counterexample if it is not.

- a. Two real numbers  $a, b$  satisfy  $a < b$  if and only if  $a < b + \epsilon$  for every  $\epsilon > 0$ .
- b. Two real numbers  $a, b$  satisfy  $a \leq b$  if and only if  $a < b + \epsilon$  for every  $\epsilon > 0$ .

**Problem 3.** For each of the following statements, write its negation. Then make a guess about whether the statement itself or its negation is true.

- a. For all real numbers satisfying  $a < b$ , there exists  $n \in \mathbb{N}$  such that  $a + 1/n < b$ .
- b. There exists a real number  $x > 0$  such that  $x < 1/n$  for all  $n \in \mathbb{N}$ .

**Problem 4.** For each  $n \in \mathbb{N}$ , define the set  $A_n = \{k \in \mathbb{N} : k \geq n\}$ . Find

$$\bigcup_{n=1}^{\infty} A_n \quad \text{and} \quad \bigcap_{n=1}^{\infty} A_n.$$