Math 301 — Functional limits

Problem 1. For each example below, give an ϵ-δ proof that $\lim_{x\to c} f(x) = L$.

a. $f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$, $c = 0$, $L = 0$

b. $f(x) = x^2$, $c = -3$, $L = 9$

Problem 2. Consider the functional limits $\lim_{x\to 0} f(x)$ for the functions below. For one of the functions, the limit exists and for the others the limit does not exist. Give an ϵ-δ proof for the one that exists and try to come up with some informal reasoning for the others.

a. $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$ (this function is called the Dirichlet function)

b. $f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}$

c. $f(x) = |x|/x$