

## Math 301 —Uniform continuity

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**Problem 1.** For each given domain  $D$  and function  $f : D \rightarrow \mathbb{R}$ , give an  $\epsilon$ - $\delta$  proof to show that  $f$  is uniformly continuous on  $D$ .

- a.  $f(x) = x^2$ ,  $D = [-4, 3]$ .
- b.  $f(x) = 1/x$ ,  $D = (2, 3)$ .
- c.  $f(x) = 1/(x - 3)$ ,  $D = (6, \infty)$ .

**Problem 2.** Suppose we wish to have a theorem that says something like:

If  $f$  is continuous on  $D$ , then  $f$  is uniformly continuous on  $D$ .

What kind of condition should be placed on the domain  $D$ ? That is, on what kind of set will a continuous function always be uniformly continuous by default. Think about examples and make a conjecture. For a challenge, try proving your conjecture using a proof by contradiction and the Bolzano Weierstrass theorem.