Math 301 —Uniform continuity

Problem 1. Consider the theorem we proved together:

> If f is uniformly continuous on D and $(x_n) \subseteq D$ is a Cauchy sequence, then $(f(x_n))$ is a Cauchy sequence.

Give the contrapositive of this statement.

Problem 2. Use the previous problem to prove that $f(x) = 1/x$ is not uniformly continuous on $(0, \infty)$.

Problem 3. State what it means for f to not be uniformly continuous on D by negating the definition.

Problem 4. Prove the following theorem:

> If there exist $\epsilon_0 > 0$ and sequences (x_n) and (y_n) on D such that $\lim_{n\to\infty} |x_n - y_n| = 0$ and $|f(x_n) - f(y_n)| \geq \epsilon_0$ for all $n \geq 1$, then f is not uniformly continuous on D.

Problem 5. Use the previous problem to prove that $f(x) = x^2$ is not uniformly continuous on $\mathbb{R}$. *Hint: choose (x_n) and (y_n) to be sequences that diverge to $+\infty$.*