

## Math 301 — Differentiability

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**Problem 1.** Prove the quotient rule.

**Problem 2.** Use the difference quotient definition to prove that  $f(x) = x$  is differentiable at any  $a \in \mathbb{R}$  and  $f'(a) = 1$ .

**Problem 3.** Use the difference quotient definition to prove that  $f(x) = x^2$  is differentiable at any  $a \in \mathbb{R}$  and  $f'(a) = 2a$ .

**Problem 4.** Let  $n \in \mathbb{N}$  and let  $f(x) = \begin{cases} x^n & x > 0 \\ 0 & x \leq 0 \end{cases}$ .

a. Let  $n = 1$ .

1. Make a conjecture about whether  $f$  differentiable at all  $a \in \mathbb{R}$ .
2. Give a piecewise formula for  $f'(a)$  at all  $a$  where you think  $f$  is differentiable.

b. Let  $n = 2$ .

1. Make a conjecture about whether  $f$  differentiable at all  $a \in \mathbb{R}$ .
2. Give a piecewise formula for  $f'(a)$  at all  $a$  where you think  $f$  is differentiable.

c. For which values of  $n$  do you think  $f$  is differentiable everywhere?