Math 301 —Differentiability

Problem 1. Sketch the graphs of each of the following functions and use intuition and past experience to make a conjecture about whether each is differentiable at the given value a. For the functions which are differentiable make a conjecture about the value of $f'(a)$.

   a. $f(x) = |x|, a = 0$

   b. $f(x) = |x^2 - 1|, a = 1$

   c. $f(x) = |x|^3, a = 0$

   d. $f(x) = \begin{cases} x^3 & x \in \mathbb{Q} \\ 0 & x \notin \mathbb{Q}, \end{cases} a = 0$

Problem 2. For each non-differentiable function in Problem 1, give a proof of non-differentiability using the sequential criterion.

Problem 3. For each differentiable function in Problem 1, give an ϵ-δ proof of differentiability. Note that your proofs should be broken into cases since the given functions are all piecewise defined.