Math 301 — Uniform continuity

Summary. A function is continuous on a domain D if it's continuous at every point in the domain. When we prove a function is continuous at a given point using an ϵ-δ proof, our choice of δ might depend on both ϵ and the domain point we're considering. However, when our choice of δ can be chosen independently of the domain point, the function is called *uniformly continuous.*

Problem 1. Try proving the following functions $f : D \to \mathbb{R}$ are uniformly continuous on the given domain D by giving an ϵ-δ proof where δ depends only on ϵ.

a. $f(x) = x^2$, $D = [-4, 3]$.

b. $f(x) = 1/x$, $D = (1, 2)$.

c. $f(x) = 1/(x - 3)$, $D = (4, \infty)$.

Problem 2. Suppose we wish to have a theorem that says something like, "If f is continuous on D, then f is uniformly continuous on D." What kind of set could D be? An open interval? A closed interval? A bounded set? Think about examples and make a conjecture. For a challenge try proving your conjecture using a proof by contradiction and the Bolzano Weierstrass theorem.