Math 301 — Introduction to sequences

Summary. We should think of proving that a sequence (a_n) converges to a value L as a challenge that must be satisfied: a challenger has given us a tiny number $\epsilon > 0$ and we must come up with a value N so that a_n is within ϵ units of L when $n > N$. That is, we need to find how far along in the sequence we must go so that we're within an ϵ- "neighborhood" of L.

Problem 1. For each sequence (a_n) and given value L, we will prove that $\lim_{n\to\infty} a_n = L$. The structure of the proofs was outlined in class: we want to find N so that $|a_n - L| < \epsilon$ when $n > N$.

a. $a_n = 1 + 2/n, L = 1$

b. $a_n = \sin(n)/n^3, L = 0$

Problem 2. Sometimes the algebra is a little more involved and there's more scratch work to do. We also might not be told the value of the limit and we must use our intuition and skills from calculus. Use the same technique and structure as the previous proofs to prove that the following sequence converges:

$$a_n = \frac{3n+1}{7n-4}.$$