Math 301 — A little more on sequences

Summary. We should think of proving that a sequence (a_n) converges to a value L as a challenge that must be satisfied: a challenger has given us a tiny number $\epsilon > 0$ and we must come up with a value N so that a_n is within ϵ units of L when $n > N$. That is, we need to find how far along in the sequence we must go so that we're within an ϵ- "neighborhood" of L.

Problem 1. Showing a sequence diverges often times requires a proof by contradiction. Try emulating the proof that $a_n = (-1)^n$ diverges (Example 4 in Section 8) to show that $a_n = \sin(n\pi/2)$ diverges.

Problem 2. Sometimes in analysis, you're interested in proving an inequality like $L \leq a$. However, that might be too inconvenient to do directly. Prove the following statement, which gives a technique for proving $L \leq a$.

Lemma. *Let $a, L \in \mathbb{R}$. If $L < a + \epsilon$ for all $\epsilon > 0$, then $L \leq a$.*

Problem 3. Let (s_n) be a sequence that converges. Show that if $s_n \leq a$ for all but finitely many n, then $\lim s_n \leq a$.